and (11),

$$\frac{P_2^1}{P_1} - 1 = \frac{\gamma M_1^2\Delta_1^2}{(M_1\Delta_1)/(1 + M_1\theta_1)}$$
$$= \gamma M_1\Delta_1(1 + M_1\theta_1) \quad (13)$$

From relations (10) and (13), we have, for the weak interaction case,

$$\frac{P_2^1}{P_1} = 1 + \gamma k + \frac{\gamma(\gamma + 1)}{4} k^2 \quad (14)$$

It may be of interest to note that the simplification of relation (11) with relation (10) will give us a series as

$$\frac{P_2'}{P_1} = 1 + \gamma k + \frac{\gamma(\gamma + 1)}{8} k^2 - \quad (15)$$

But this series, in relation (15), contains alternatingly negative and positive terms after the third term As such, this procedure may not be good, though the approximation can easily be made up to the third term

Conclusions

From relations (12) and (14), it can be seen that the pressure distribution in relation (5) is having a very good approximation of the tangent-wedge pressure distribution Also, it may be of interest to note that relations similar to relations (9) and (10) can be derived from relation (2) as well From the present note and the note in Ref 3, it can be seen that the pressure distribution of Pai in Ref 1 is well modified to bring the results very near to the tangent-wedge approximation

References

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An Analysis of the Yawing Motion of a Rocket with a Varying Mass

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Nomenclature

m = mass of the rocket as a function of time, slugs
m_0 = mass of the rocket at ignition, slugs
m_b = mass of the rocket at burnout, slugs
c = constant related to the burning rate of the propellant, per sec
t = time, sec
V = horizontal velocity of the rocket, fps
S = cross sectional area of the rocket, ft²
T = thrust lb
k = radius of gyration, ft
r = distance from center of mass to nozzle exit, ft
l = length of the rocket, ft
C_d = aerodynamic drag coefficient
C_m = aerodynamic restoring moment coefficient
ρ = atmospheric density slugs/ft³
φ = angle of yaw, rad

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The angular oscillation about a horizontal flight path, experienced by a fin-stabilized rocket during the burning period, is known to have an increasing period and a decreasing amplitude This behavior is clearly due to the decreasing moment of inertia as the rocket loses mass and to the increasing magnitude of the restoring moment as the velocity of the rocket increases The authors have presented a simplified analysis which demonstrates the yaw behavior in a closed-form solution

The flight path of the rocket is horizontal, and the aerodynamic coefficients are assumed constant Although the mass is a function of time, it is assumed that the radius of gyration and the center of mass with respect to the vehicle remain unchanged The rate at which the propellant is consumed is constant, and the mass is represented as

$$m = m_0(1 - ct)$$

The equation of motion for the center of mass of the rocket is

$$m_0(1 - ct)(dV/dt) + \tfrac{1}{2}C_D\rho SV^2 = T$$

where the thrust is a constant This equation, of the Ricatti type, may be integrated by separation of variables to yield

$$V(t) = (T/\tfrac{1}{2}C_D\rho S)^{1/2}[C_0 + (1 - ct)^B]/[C_0 - (1 - ct)^B] \quad (1)$$

where

$$B = 2(\tfrac{1}{2}C_D\rho ST)^{1/2}/cm_0$$
$$C_0 = [V_0 + (T/\tfrac{1}{2}C_D\rho S)^{1/2}]/[V_0 - (T/\tfrac{1}{2}C_D\rho S)^{1/2}]$$

and V_0 is the horizontal velocity at the time of ignition

The equation describing the yaw is derived in Ref 1 and may be written as

$$m_0k^2(1 - ct)\ddot{\varphi} + (r^2 - k^2)cm_0\dot{\varphi} + \tfrac{1}{2}C_m\rho lSV^2\varphi = 0 \quad (2)$$

where the dot signifies the derivative with respect to time The angle of yaw is assumed small so that $\sin\varphi \approx \tan\varphi \approx \varphi$ Substituting for the velocity from Eq (1) and dividing by the leading coefficient gives

$$\ddot{\varphi} + \frac{c(r^2 - k^2)/k^2}{(1 - ct)}\dot{\varphi} + \frac{C_mlT/C_Dm_0k^2}{(1 - ct)}\left[\frac{C_0 + (1 - ct)^B}{C_0 - (1 - ct)^B}\right]^2\varphi = 0 \quad (3)$$

For small rockets (final mass to initial mass ratios greater than 80%) the effect of the second term in Eq (3) is small in the final result; therefore, the coefficient is replaced by a constant average value $2K$ defined as

$$2K = \tfrac{1}{2}[c(r^2 - k^2)/k^2][(m_0 + m_b)/m_b]$$

The equation is then rewritten as

$$\ddot{\varphi} + 2K\dot{\varphi} + K_1^2[V^2(t)/(1 - ct)]\varphi = 0 \quad (4)$$

where

$$K_1^2 = \tfrac{1}{2}C_m\rho lS/m_0k^2$$

Furthermore, the variable coefficient of φ is approximated as

$$V^2(t)/(1 - ct) \simeq V_0^2(1 + t/T_b)^2$$

where

$$t_b/T_b = (1 + \Delta V/V_0)/(1 - \Delta m/m_0)^{1/2} - 1$$

and where t_b is the time of burnout, $\Delta V/V_0$ is the fractional velocity increase during the burning period, and $\Delta m/m_0$ is the fractional mass decrease over the same time Introducing the approximation into the equation yields

$$\ddot{\varphi} + 2K\dot{\varphi} + K_1^2V_0^2(1 + t/T_b)^2\varphi = 0$$

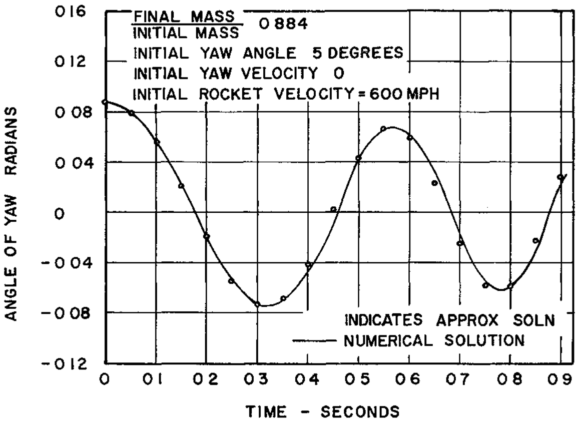

Fig 1 Yaw angle as a function of time, comparing the approximate solution with a numerical solution of the differential equation

Defining a nondimensional time

$$\tau = 1 + t/T_b$$

reduces the equation to

$$\varphi'' + 2KT_b\varphi' + K_1^2V_0^2T_b^2\tau^2\varphi = 0 \tag{5}$$

where the prime denotes differentiation with respect to τ By letting

$$\varphi(\tau) = e^{-KT_b\tau}\theta(\tau)$$

the first derivative in Eq (5) is removed, giving

$$\theta'' + K_1^2V_0^2T_b^2(-K^2/K_1^2V_0^2 + \tau^2)\theta = 0 \tag{6}$$

For the size of rocket studied, a representative value of $(K/K_1V_0)^2$ is $(3/V_0)^2$, where V_0 is the initial velocity in feet per second Since the smallest value of τ is one, the factor $(3/V_0)^2$ may be neglected if the rocket has an appreciable initial velocity In the numerical computation, the rocket was given an initial velocity of 600 mph, and hence the factor $(3/V_0)^2$ is neglected In this case, Eq (6) becomes

$$\theta'' + K_1V_0^2T_b^2\tau^2\varphi = 0 \tag{7}$$

The solution to Eq (7) is given in terms of Bessel Functions:

$$\theta = C\tau^{1/2}J_{1/4}(\tfrac{1}{2}K_1V_0T_b\tau^2) + D\tau^{1/2}Y_{1/4}(\tfrac{1}{2}K_1V_0T_b\tau^2)$$

where the constants C and D are determined from the initial conditions The solution for yaw is, therefore,

$$\varphi(t) = \tau^{1/2}e^{-KT_b\tau}[CJ_{1/4}(\tfrac{1}{2}K_1V_0T_b\tau^2) + DY_{1/4}(\tfrac{1}{2}K_1V_0T_b\tau^2)] \tag{8}$$

Assuming an initial velocity such that $K_1V_0T_b\tau^2 > 10$, the argument of the Bessel Functions becomes large enough to allow approximation by trigonometric functions For the rockets studied, the factor $K_1V_0T_b\tau^2$ ranged from 10–40 for an initial velocity of 600 mph In this case,

$$\varphi(t) = C_1\tau^{-1/2}e^{-KT_{bb}\tau}\cos(\tfrac{1}{2}K_1V_0T_b\tau^2 - D_1) \tag{9}$$

where C_1 and D_1 are determined from the initial conditions Equation (9) exhibits, in closed form, the property that the amplitude of the oscillation decreases as the factor $\tau^{-1/2}e^{-KT_b\tau}$, and that the frequency increases proportionally to the nondimensional time

In Fig 1 the approximation given by Eq (9) is compared to a numerical solution of the original equation for yaw [Eq (3)] demonstrating that Eq (9) is a satisfactory approximation for the case studied

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Magnetohydrodynamic Hypersonic Viscous Flow Past a Blunt Body

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In this paper the equations governing the hypersonic flow about a magnetized body are solved numerically The solutions obtained illustrate the effects of Reynolds number and magnetic parameter on the structure of the applied magnetic field The results show that, for magnetic parameters larger than a critical value, numerical solutions do not exist A critical magnetic parameter is considered to result from an assumption of a given magnetic field at the shock By increasing the conductivity, and thereby the induced magnetic field, the applied field at the shock disappears This, in turn, violates the basic assumption of a finite field at the shock It is concluded that the applied magnetic field can only be defined at the body where it is a natural characteristic of the problem

Introduction

THE mathematical complexity of the problem of hypersonic flow about a magnetized body is great even if the investigation is restricted to the local similarity solutions For computational simplification, Bush and Wu investigated the problem by considering the given magnetic field strength at the shock wave [1, 2] Bush found that numerical integration becomes impossible for the inviscid case when the value of the magnetic parameter exceeds a certain critical value This paper extends Wu's study of the viscous case[2] to a much wider range of the parameters by using digital computer techniques Computational difficulties similar to those found by Bush were encountered This paper will report some results and at the same time provide a physical interpretation for the nonexistence of numerical solutions for magnetic parameters higher than the critical value, which depends upon the value of the viscous Reynolds number

Local-Similarity Solution

The assumptions used here are those used in Ref 2 The vehicle is assumed to be moving at a speed (Mach >15) sufficient to cause a significant amount of shock ionization of the gas behind the detached shock wave In addition, the fluid is assumed to be incompressible, viscous, and in steady state The applicable equations are as follows:

$$\nabla \cdot \mathbf{v} = 0$$
$$\nabla \times [(\nabla \times \mathbf{v}) \times \mathbf{v} - (\mu_e/4\pi\rho)(\nabla \times \mathbf{H}) \times \mathbf{H}] = \nu\nabla^2\nabla \times \mathbf{v} \tag{1}$$
$$\nabla \cdot \mathbf{H} = 0$$
$$\nabla \times [\nabla \times \mathbf{H} - 4\pi\sigma\mu_e\mathbf{v} \times \mathbf{H}] = 0$$

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